Design optimization of the CADRE Magnetorquers

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Contents

Nomenclature

M	Magnetic dipole moment magnitude, A-m^2
N	Number of turns of wire
i	Electric current
A	Area of the coil
$\hat{n}$	Normal vector to the coil
τ	Torque induced on coil of wire
K	Gain factor
k	Iteration number
H_k	Hessian matrix for iteration k
d_k	Search direction for iteration k
x_k	Current point for iteration k
γ_k	Step size for iteration k
s_k	Parameter used to simplify BFGS update expression
y_k	Parameter used to simplify BFGS update expression
μ	permeability of the core
μ_0	Permeability of free space
μ_r	Relative permeability of core material
r	Core Radius
V_{bus}	Voltage supplied to the torquer from the spacecraft bus
R	Resistance of the wire
N_d	Demagnetizing factor
l	Length of torque rod
R_{Cu}	Resistivity of Copper
a_w	Gauge of Copper wire
l_w	Length of wire
ρ_{core}	Density of Iron Ferrite core
ρ_{Cu}	Densit of Copper

1 Problem Introduction

CubeSats are small, inexpensive research satellites that are usually launched as secondary payloads. The CubeSat standard, proposed in 1999 by Standford University and Cal Poly, specifies a 10 cm x 10 cm x10 cm per unit cube as a vehicle to support academic education and frequent access to space. This platform is perfect for university-level exploration due to its relative low cost and short design life cycles. In part, this is a result of the wide use of commercial off-the-shelf electronics. CubeSats provide students with opportunities to have real spaceflight hardware and software experience from their time in school.

The Michigan Exploration Lab (MXL) currently has three CubeSats in orbit with several more proposed projects in the pipeline. MXL has already proposed CADRE, the CubeSat investigating Atmospheric Density Response to Extreme Driving, which will be one of the first CubeSat missions funded through the National Science Foundation (NSF). Most flight hardware will be built in-house in Ann Arbor, except for the primary science payload. CADRE will carry WINCS, the Wind Ion Neutral Composition Suite, in order to record the global dynamics of the thermosphere and ionosphere. Armada, the follow-on mission to CADRE, will consist of 48 CADRE satellites in 6 orbital planes. The simultaneous data measurements collected by Armada will improve tracking of orbital objects for both situational awareness and orbital collision prevention and improve GPS reception in polar areas.

The success of the CADRE mission hinges on a reliable, capable Attitude Determination and Control System (ADCS). This report will explain the motivation for magnetic torquers and their requirements, detail the optimization method for our proposed torquer design, and discuss the design space and future work.

1.1 Active Control on CubeSats

Attitude control of a CubeSat can be accomplished two ways: actively and passively. Passive stabilization constitutes permanent magnets, gravity gradient, or differential drag. It is the most popular control method for the CubeSat platform as it requires minimal to no on-board power or processing. It's simplicity also ensures a robust system design, however it is difficult to meet high pointing accuracy requirements with this form of stabilization. Active control increases the complexity of the mission but can ensure high pointing accuracies. Magnetorquers, momentum/reaction wheels, reaction control thrusters, control moment gyros, and spin stabilization are examples of active control systems and actuators.

In order for CADRE to be successful, the principal investigator Professor Aaron Ridley has dictated a 1 degree pointing accuracy requirement (2 degree cone) with 0.1 degrees of attitude knowledge. This will ensure that the heavy ions enter WINCS with a known (or deterministic) relative trajectory. To achieve this level of accuracy, CADRE will employ three orthogonal momentum wheels (for control), a three-axis magnetic control system (for wheel desaturation), and a full suite of determination sensors including: a star tracker, two coarse sun sensors, three high precision rate gyroscopes, photodiodes, and magnetometers [?].

If CADRE flew today, it would be the most advanced CubeSat ever launched. The only commercial packages being purchased are the Sinclair momentum wheels, a Boeing Star Tracker, the sun sensors and the WINCS instrument designed by the Naval Research Laboratory (NRL). However, the three magnetorquers are being designed by the Michigan Exploration Laboratory, which reduces mission costs, increases the technology heritage of the MXL, and gives students hands-on experience in design and assembly.

1.2 Review of Magnetorquers

A magnetorquer consists of a coil of wire that produces a rotational torque when an electric current is passed through the coil. This is similar to an inductor. Unlike inductors, which are wound to produce maximum inductance, magnetorquers are wound to provide maximum rotational torque on the coil. For CADRE, the torquers will provide momentum dumping for the reaction wheels and will work on three axes. Active magnetic control has several important advantages:

- Low power consumption and low mass
- Suitable for restricted volumes due to custom design possibility
- No moving elements
- Slow transient response due to low torque production capacity
- Uncertainty in magnetic field model and errors in measurements can lead to unstable control. Even the most accurate models (such as IGRF) are only approaching reality.

The magnetic dipole moments produced by the magnetorquers are proportional to the electric current running through them. The magnetic dipole, M, produced by an air core is defined by:

$$\vec{M} = NiA\hat{n} \tag{1}$$

Here, N refers to the number of turns, i is the electric current, A is the area of the coil, $\hat{n}$ is the normal vector of the coil. If the magnetic moment vector of the spacecraft is not aligned with the Earths magnetic field, a torque is induced on the coil of wire defined by the cross product:

$$\vec{\tau} = \vec{M} \times \vec{B} \tag{2}$$

1.3 Vacuum Core vs. Solid Core

Introducing a ferrite core in the magnetic torquer increases the dipole moment of the solenoid by up to 3000 times (gain factor K=100-3000). To reach the same dipole moment with an air core magnetic actuator, you need to either increase the enclosed area/number of turns (and thus mass) or increase the current flowing through the windings (and thus power). The previous equation for magnetic moment is modified to:

$$\vec{m} = KNi\vec{A} \tag{3}$$

Here, K depends on the length/diameter shape factor and permeability of the material.

1.4 History of MXL Magnetic Control

The Michigan Exploration Lab has successfully launched three CubeSats in the past three years (RAX, RAX-2 and M-Cubed). All three used AliNiCo-5 permanent magnets for attitude stabilization. M-Cubed-2, a follow on mission, to be delivered to the launch provider in May, employs a single air-core magnetorquer. The M-Cubed-2 magnetorquer was *not* optimized for magnetic moment, power or mass. It was intended as a pathfinder to demonstrate flight fabrication processes. The design was chosen by examining the design space and subjectively selecting a configuration that satisfied requirements.

The coil was manufactured from white acetal Delrin, shaped by Professor Washabaugh's laser cutter. The coil was designed in layers less than 0.25 inches (the maximum laser cutter height). After the mount was assembled, the 30 AWG wire was hand-wound to 193 turns for a designed magnetic moment of 0.36 $A - m^2$. After winding, the coil was dunked in a 3M ScotchCast solution that rigidly locks the wire down, while also adding additional environmental protection. This same manufacturing procedure will be repeated for the CADRE torque rods. It is important to understand the physical and mechanical limitations before engaging in optimization.

2 CADRE Requirements

When CADRE was first explored in the AEROSP 483 class of 2011, the team base-lined the commercially available CubeTorquer from ISIS. The estimated magnetic moment, mass and power consumption of the rod was about 0.2 $A - m^2$, 30g and 200 mW. All of the power, mass, and data budgets were evolved from those specifications. After the magnetorquer fabrication procedure was proven on M-Cubed-2, the decision was made to fabricate the CADRE torquers in-house. Thus, the requirements imposed on the custom torquers are

1. Mass shall not exceed 30 g per axis
2. Power consumption shall not exceed 200 mW per axis
3. All three coils must fit comfortably in the CADRE 3U form factor
4. The magnetic moment shall pass all simulations done using the control algorithms implemented in the MXL's Matlab/Simulink/C++ propagator.

To date, the simulations have been run using a magnetic dipole moment of exactly 0.05 $A-m^2$. The magnetorquers are only *required* to desaturate the wheels. However, in the event of a wheel failure, it would be ideal for the magnetorquer to be capable of re-orienting the satellite. For this to be feasible, the dipole strength of the magnetorquers must be able to overcome the environmental disturbance torques acting on CADRE. Table 1 summarizes the peak order of magnitude disturbances that CADRE must over come. The derivations can be found in Appendix A

Torque Type	$\mathrm{N}\cdot\mathrm{m}$
Residual Dipole Torque	4.00×10^{-7}
Aerodynamic Torque	6.97×10^{-9}
Gravity Gradient	1.75×10^{-9}
Solar Pressure	1.64×10^{-9}
RMS Sum	4.1×10^{-7}

Table 1: Estimated external torques for the CADRE CubeSat

These results are corroborated with the conclusions from AAU Sat and the CalPoly PolySat.

2.1 Literature Review

As a sanity check, a brief literature review was performed to compare the ideal magnetic for CADRE (0.2 $A-m^2$) with other magnetorquers that have flown on CubeSats in the past. Table 2 summarizes as many designs as could be found freely on the web. It is clear that 0.2 $A-m^2$ is a reasonable design choice and has heritage on previous missions.

Mission	Type	Magnetic Moment (A-m^2)	Power (mW)	Mass (g)	Size	Notes
AAUSat	Air Core		122	20	8cmx9cm	X-Sec A=10mm^2, C=356mm, R=100 ohms, Vbus=10V
AAUSat-3	Iron Core	0.03	6.8	19	200	-
CanX 2	3 Air Cores	0.1	40	100	XXX	5-35°C, built own winder
COMPASS-1	Air Core	0.085	26	19.2	400 turns	-
U Toronto GNB	Air Core	0.19	26	104	210 turns	-
GNB (2)	XXX	0.19	21	108	235 turns	-
Illinois, ION	Air Core	0.149	100	XXX	1500 turns	1.32e-8 m^2 X-sectional area, f Belden heavy armored poly-thermaleze 38 AWG
Illinois, TinySat	PCB Traced	XXX	114mA	XXX	120 loops	R=96.3Ω, 0.0007 in wire
CalPoly PolySat	PCB Traced	-	300mA	-	54 turns	0.1503 m^2
Cute 1.7	3 Air Core	0.15	91	5	58.5 x 78.3 mm	2U Cubesat, 1 coil, 13mA drive current
SwissCube	3 Air Core	0.0285	-	-	-	Bdot and LQR
ISIS	Alloy Core	0.2	200	30	7 cm x 1 cm	-35 to 75°C, 1200
CubeTorquer	Iron Core	0.2	209	22	6 cm x 1 cm	Supra50 core, 1200 Euro

Table 2: Summary of CubeSat magnetorquers

3 Mechanical Design

The three CADRE magnetorquers were designed to integrate easily into the CADRE bus structure without taking up too much volume. As a first step, a variety of designs were surveyed to see what would be the best fit for the mission architecture, several of which can be seen in Figure 1.

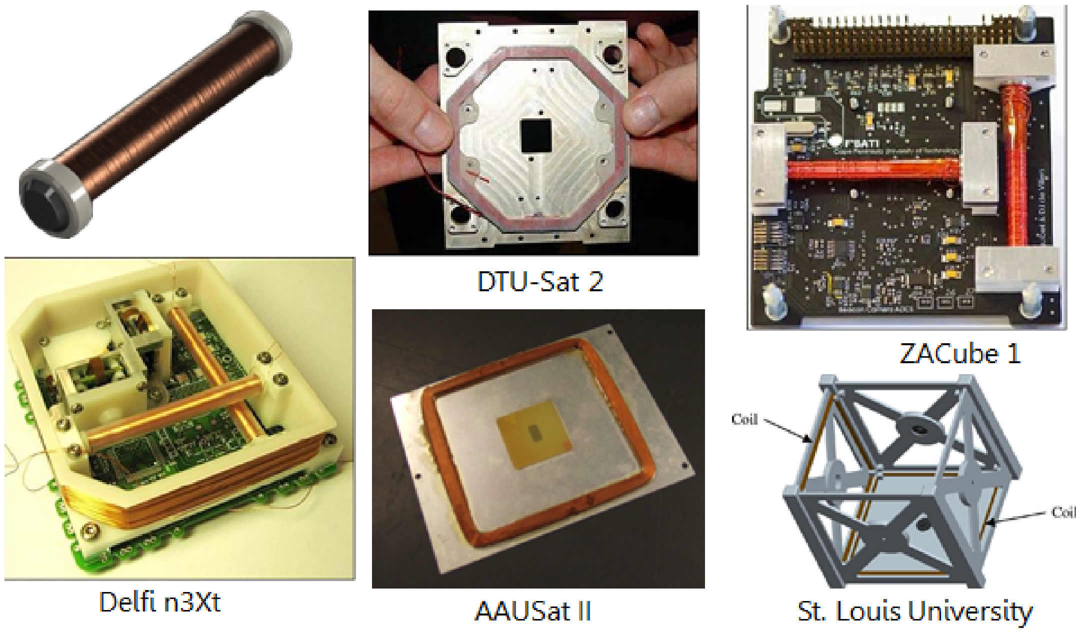

Figure 1: An survey of various mechanical designs for torque rods

We also considered inscribed magnetorquers, which are copper traces inscribed in PCB layers. However, the cost, complexity in routing, and lack of heritage ruled it out. After much consideration, the design that we settled on consists of two iron core X-Y torque rods and an air core z-axis coil. In this way, all three magnetorquers can fit onto the same PCB called the Torquer Control Board (TCB). The TCB will also be designed to be the "connector hub" for a variety of connectors in the ADCS bay. Figure 2 indicates the proposed mechanical design for the X-Y torque rods.

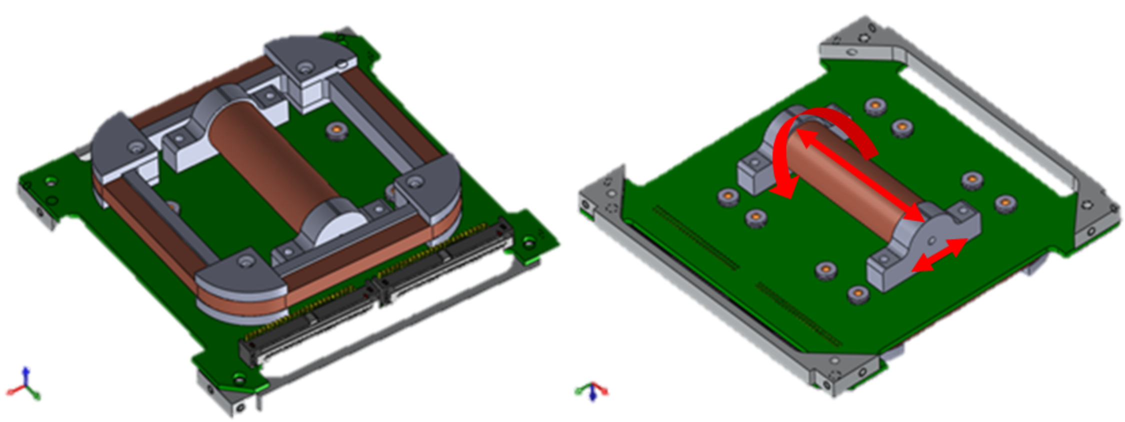

Figure 2: Proposed mechanical design with free variables called out (core length, diameter, turn count and wire gage)

Optimization techniques were employed to size the two iron core X-Y torque rods. There were four free variables open in our design: core length, core diameter, number of turns, and wire gauge. All other variables (wire length, mass, current, power, magnetic moment) can be derived from these four. In the scope of the following analysis, the wire gauge has been fixed as 40 AWG for the X-Y torque rods.

4 Optimization Method

The method for determining the optimal magnetic torquer design for the X-Y axes is described in the following subsections. The design space focuses on maximizing the magnetic dipole moment of the torquers while constraining the number of turns, radius of the core, and the length of the torquer. The Matlab *fmincon* was used to find the values of these variables that both satisfied the constraints and maximized the magnetic dipole moment expression.

4.1 *fmincon*

Matlab contains an Optimization Toolbox that contains functions of typical optimization algorithms. The function used for the optimization of the CADRE magnetic torquers was *fmincon*. The command solves constrained nonlinear optimization problems by finding the constrained minimum of a scalar function of multiple variables based on a provided initial estimate [?]. The command for *fmincon* includes the function to be optimized, the initial estimate of the maximizer, and the constraints. The Matlab command for the sizing of the magnetic torquers was as follows:

[x,fval]=fmincon('core',[100; 0.3e-2;5e-2; 100],[],[],[],[],[],[],'mycon',options)

core was the name of the Matlab function created to maximize the magnetic dipole moment (in units of A-m^2); *mycon* was a function that contained the constraints on the variables; *options* designated the maximum function evaluations, tolerances of the function, tolerances of the maximizer, and iterations for the optimizer; and [100; 0.3e-2;5e-2] was the intial guess (100 turns of the wire, 3 mm radius of the core, and 5 cm length of the core).

4.1.1 Algorithm

fmincon uses a sequential quadratic programming (SQP) method which essentially steps through a quadratic programming "subproblem" for each iteration. During each iteration, an estimate of the Hessian of the Lagrangian is updated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula [?]. The BFGS method is a Quasi-Newton method that can be considered a generalization of the secant method [?], and was discussed in lecture Module 4. Essentially, the method depends on the Hessian matrix, H_k, the point x_k, the step-size γ_k, and the search direction d_k. The BFGS steps are described as follows [?]:

1. Specify an initial H_0 and x_0. If the user does not specify H_0, then the BFGS method uses the default $H_0 = I$

2. For each iteration (k = 0, 1, 2, ...).
   (a) End if x_k is optimal (when the gradient is zero, which is the necessary condition for optimality).
   (b) Solve Equation 4 to determine the search direction d_k.
   (c) Use a line search method to determine the step size γ_k >0 (instead of $\gamma = 1$), which ensures that the Wolfe conditions are satisfied for each step of the iteration. Typically, a mixed quadratic/cubic line search procedure is used to determine γ_k.
   (d) Update the point, x_{k+1} using Equation 5.
   (e) Compute H_{k+1} using the BFGS update, Equation 6.

$$f''\left(x_k\right) d_k = -\nabla f\left(x_k\right) \tag{4}$$

$$x_{k+1} = x_k - \left[f''\left(x_k\right]^{-} 1 \nabla f\left(x_k\right) \tag{5}$$

$$\begin{array}{rcl} H_{k+1} & = & H_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{(H_k s_k)(H_k s_k)^T}{s_k^T H_k s_k} \\ y_k & = & \nabla f\left(x_{k+1}\right) - \nabla f\left(x_k\right) \\ s_k & = & \left(x_{k+1} - x_k\right) \end{array} \tag{6}$$

The BFGS is a rank two update formula that maintains symmetry and positive definiteness of the Hessian, which ensures that the search direction, d_k, is always a descent direction. A line search is then performed using a merit function and the subproblem is solved using an active set strategy [**?**].

4.1.2 Limitations

There are several limitations of the *fmincon* function [**?**]:

- *fmincon* can only be used for real variables.
- The function to be minimized/maximized and the constraints must be continuous.
- *fmincon* may only solve the local solution.
- If the problem is infeasible, *fmincon* will attempt to minimize the maximum constraint value.
- The objective and constaint functions must be real-valued (i.e. they do not return complex values).

4.2 Physical Equations and Variables

The expression for the magnetic dipole moment contained in *core* is described by Equation 7 [?].

$$\boxed{f(x) = M = \frac{\pi r^2 N V_{bus}}{R}\left(1 + \frac{\mu_r - 1}{1 + (\mu_r - 1)N_d}\right) \rightarrow max} \tag{7}$$

Where r is the radius of the core of the torquer and is one of the variables defined by x; N is the number of turns of the wire and is one of the variables defined by x; V_{bus} is the voltage supplied to the magnetic torquer by the spacecraft bus, a constant 8.2 V; μ_r is the relative permeability of the core material, a constant 2000; N_d is the demagnetizing factor described by Equation 9; R is the resistance of the wire described by Equation 10 [?].

$$\mu_r = \frac{\mu}{\mu_0} \tag{8}$$

$$N_d = \frac{4[\ln\left(\frac{l}{r}\right) - 1]}{\left(\frac{l}{r}\right)^2 - 4\ln\left(\frac{l}{r}\right)} \tag{9}$$

$$R = \frac{2\pi r N R_{Cu}}{a_w} \tag{10}$$

The resistance of the wire depends on the resistivity of Copper, $R_{Cu} = 1.55\text{x}10^{-8}\ \Omega - m$, and the gauge of the Copper wire, $a_w = 7.97\text{x}10^{-9} m^2$. The variables of x are constrained within the function *mycon*. The constraints for this problem set-up are inequality constraints, described by Equation 11, where ρ_{core} is the density of the core (Iron Ferrite), 8.74x10^3 kg/m^3; ρ_{Cu} is the density of the Copper wire, 8.93x10^3 kg/m^3; and l_w is the length of the wire described by Equation 12.

$$\begin{cases} h_1(x) = \rho_{core}\pi r^2 l + a_w l_w \rho_{Cu} - 0.03 \leq 0 \\ h_2(x) = \frac{V_{bus}^2}{R} - 0.2 \leq 0 \\ h_3(x) = N - 10,000 \leq 0 \\ h_4(x) = r - l \leq 0 \end{cases} \tag{11}$$

$$l_w = 2\pi r N \tag{12}$$

There are four inequality constraints for sizing the magnetic torquers:

1. $h_1(x)$ constrains the mass to be less than 30 grams to meet the mass requirement of the magnetic torquer.

2. $h_2(x)$ constrains the power consumed by the magnetic torquer to be less than 200 mW to meet the requirements.

3. $h_3(x)$ constrains the number of turns to be less than 10,000, the maximum number deemed reasonable to manufacture.

4. $h_4(x)$ constrains the radius of the core to be less than or equal to the length of the torquer, which defines the reasonable design space region.

4.3 Comments on Design Space

Although the proposed problem may seem like a very straight forward minimization problem (maximize magnetic moment subject to mass and power constraints), the design space is relatively complex. As shown in Figure 3, the function is not strictly convex. In fact, to get reasonable solutions, we constrained *fmincon* to only consider the designs with length greater than or equal to rod radius.

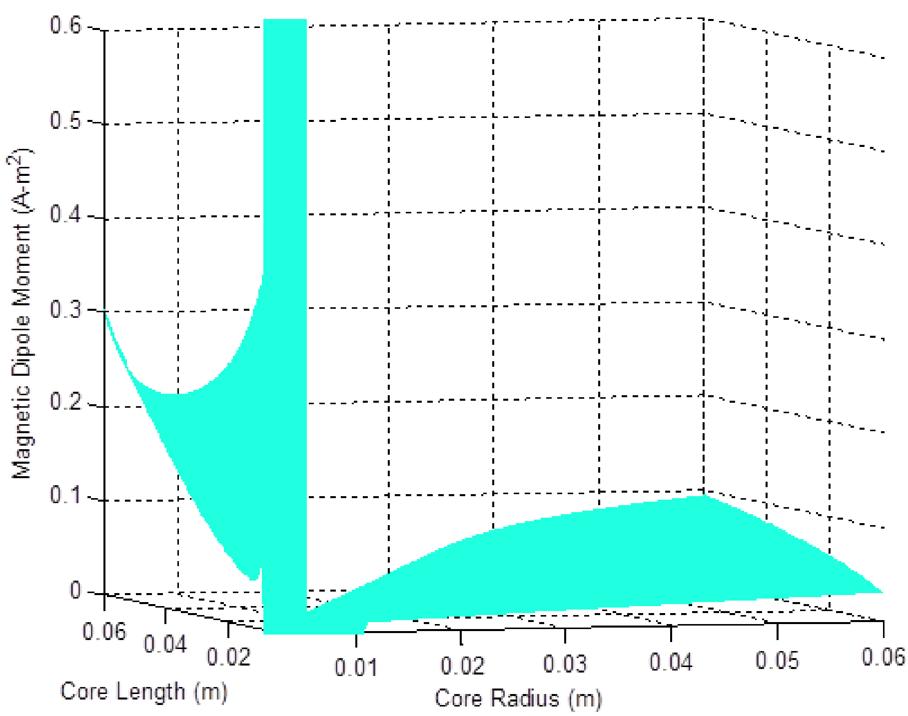

Figure 3: The function is not strictly convex.

It should be noted that most of the magnetic torquer mass is from the core/mount and not from additional wire length since the gauge of the Copper wire has a very low mass per unit length. Additionally, several trials were run to determine whether or not altering the constraints of system would yield more optimal results in terms of mass and power (CubeSats are greatly constrained in mass and power due to the small platform). Table 3 provides an sample of the many trials that were run and the yielded results. The proposed torquer design is ROW 1 in the Table.

CONSTRAINTS			RESULTS					
Power	**Mass**	**Length (cm)**	**DIPOLE**	**Mass**	**Power**	**Turns**	**Radius (mm)**	**Length (cm)**
200mW	30g	5cm	0.217	30g	200mW	7.66k	3.59mm	5cm
100mW	30g	5cm	NO Solution					
100mW	50g	5cm	No Solution					
150mW	50g	5cm	0.193	50g	150mW	7.416k	4.9mm	5cm
100mW	50g	3cm	0.107	49g	100mW	9.9k	5.5mm	3cm
300mW	50g	2cm	0.1064	32g	298mW	2.94k	6.3mm	2.2cm
200mW	30g	2cm	0.0707	18.2g	200mW	8.404k	3.3mm	2cm

Table 3: Trials of different constraints defined in *mycon*

From the trials it was clear that smaller magnetic dipoles would not save much mass or power for the system. This alludes to the complexity of the design space. Figure 4 from the M-Cubed-2 design simulations shows the same trends that are applicable to CADRE.

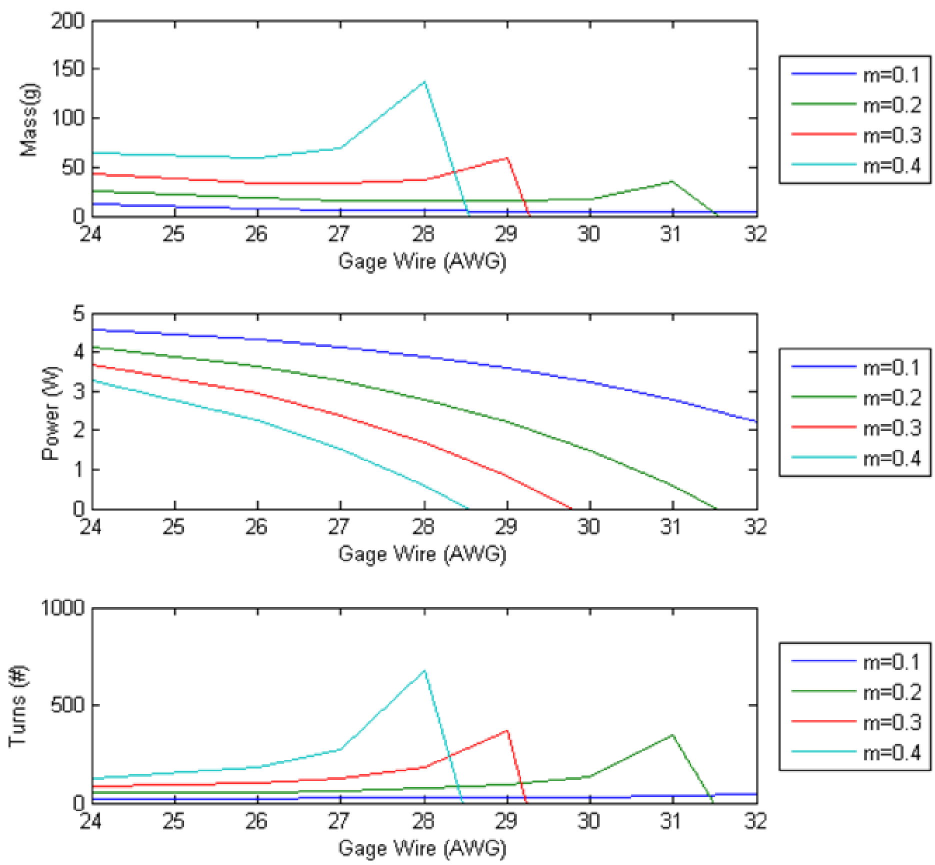

Figure 4: The trends for the M-Cubed-2 air core also apply to the design space of CADRE.

Increasing the wire gauge (smaller area) reduces the power (higher resistance). However, for the same magnetic moment, a smaller qire gauge requires more turns → which requires an increased wire length → increases resistance → decreases current → requires more turns. Turn count and mass reach infinity → infeasible range has no solution. This behavior was similarly observed in the CADRE optimization script. There were some constraints that did not have a solution. One must realize that simply decreasing the required magnetic dipole moment may not reduce power/mass and make such designs feasible!!

5 Proposed Design

The proposed design for the magnetic torquers was the result of the first trial constraints listed in Table 3. The constraints were that the power had to be less than 200 mW, the mass less than 30 g, and the length of the torquer had to be less than 5 cm. These constraints resulted in a magnetic dipole moment of approximately 0.22 A-m^2 based on a torquer that had 7.66x10^3 turns, a core radius of 3.59 mm where the power, mass, and torquer length constraints (h_1 - h_3) were active. $h_4(x)$ was not active, but the constraint was met.

The z-axis air core magnetorquer will be nearly identical to the M-Cubed-2 flight magnetorquer, only with a smaller wire gauge. The optimization tools demonstrated in this paper will be adapted to meet requirements in this third axis. Since the z-axis lacks a ferrite core, some of the assumptions used to derive Equation 7 break down, which would require a new Matlab script to be written. This will be accomplished in early May.

5.1 Z-Axis Design

Reference "mcubed_magnet.pdf"

6 Fabrication Procedure

for scotchcast dunking etc review "MCubed-2 Fabrication Procedures.pdf". A couple of new things. need to learn how to use the winder.

Figure 5: The trends for the M-Cubed-2 air core also apply to the design space of CADRE.

winder settings

7 Contributions

I identified the optimization problem and developed the method for finding the solution in Matlab. Jessica Arlas was able to review my code and made a couple of improvements on the constraint functions. Jessica also discussed the opimization method and commented on the design space. I completed the rest independently.

References

[1] BFGS - Gradient Approximation Methods. Butler's Math.

[2] fmincon, The Hebrew University, Jerusalem.

[3] Optimization Toolbox: fmincon. Weizmann Institute of Science.

[4] Mohamad Fakhari Mehrjardi. Design and Manufacturing of a Research Magnetic Torquer Rod. *Contemporary Engineering Sciences, Vol. 3, 2010, no. 5, 227-236*, 3:227–236, 2010.

[5] Wiley J. Larson & James R. Wertz. Space mission analysis and design. -, 1:1, 1999.

Appendices

A Order of Magnitude Torque Disturbances

The worst case torque generated by environmental disturbances is used to size the permanent magnet/magnetorquer. M-Cubed must overcome the worst case torque in order to control the satellite in any given situation. Although it is unlikely that M-Cubed-2 will be exposed to the worst case torque in space (all disturbances acting in the same direction), we still sum the torques from the [residual dipole, gravity gradient, atmospheric drag, and solar radiation pressure].

Magnetic Disturbance Torque
Magnetic dipoles stem from two sources. First, they can occur transiently from the on-board electronics–especially high-current modules such as radios. Also, the structure of the spacecraft may contain a residual dipole that can also be a source of unwanted disturbance angular moments. As a rule of thumb, residual dipole moment on spacecraft is around 0.01 A-m^2.

$$\vec{B_{Earth}} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1 + sin(\lambda)^2} \tag{13}$$

Where $\frac{\mu_0 M}{4\pi}$ is the magnetic dipole moment from the Earth, R is the distance to center of the Earth, λ is the magnetic latitude.

$$T_{res} = \vec{M_{res}} \cdot \vec{B_{Earth}} = 0.01Am^2 \cdot 4 \times 10^{-5}T = 4 \times 10^{-7}Nm \tag{14}$$

Where $\vec{M_{res}}$ is the residual dipole of the spacecraft.

Gravity Gradient

$$T_g = (I_{max} - I_{min})3n_{max}^2 = (0.0005kg \cdot m^2)3(0.00108\frac{rad}{s})^2 = 1.75 \times 10^{-9}Nm \tag{15}$$

Where I is the principle moment of inertia in that axis, and n is the angular rate of orbit.

Atmospheric Drag Force

$$F_d = \frac{1}{2}\rho v^2 C_d A(\vec{N} \cdot \vec{D}) \tag{16}$$

Where ρ is the atmospheric density at the orbit altitude, $\vec{N}$ is the normal vector of the body face, C_d is the coefficient of drag, A is the cross sectional area, and v is the velocity relative to atmosphere.

Atmospheric Drag Torque

$$\tau = P \times F_d = \frac{1}{2}(4.89 \times 10^{-13}\frac{kg}{m^3})(7550\frac{m}{s})^2 2.5(0.01m^2)0.02m = 6.97 \times 10^{-9}Nm \tag{17}$$

The torque then, is generated by the cross product of P, the lever arm, which is the distance between center of aerodynamic pressure and geometric center (<2cm by CubeSat requirements) and the atmospheric drag force, F_d.

Solar Radiation Pressure

$$T_S = \frac{\phi}{c^2} A(1+Q)(N \cdot S)d = \frac{1367 \frac{W}{m^2}}{3 \times 10^8} \frac{m}{s} 0.01m^2(1+0.8)0.2m = 1.64 \times 10^{-9} Nm \tag{18}$$

Where ϕ is the universal solar constant; c, speed of light; Q, panel reflectance; S, sun vector (gives angle of incidence)

B Matlab Optimization Simulation

```
%University of Michigan
%Description:
% This program maximizes the magnetic moment for the CADRE x-y ferrite
% magnetorquers subject to mass and power constraints. The free
% variables are core length, core diameter, wire length and wire gauge.

clear all
close all
clc

options = optimset('MaxFunEvals',10^3, 'MaxIter', 10^3, 'TolFun', ...
    10^-15, 'TolX', 10^-15);
[x,fval]=fmincon('core',[100; 0.3e-2;5e-2; ...
    100],[],[],[],[],[],[],'mycon',options)
display('hi')
```

```
function [ M_total ] = core( x )
%UNTITLED3 Summary of this function goes here
%   Detailed explanation goes here

N=x(1);
r=x(2);
l=x(3);
%Rseries = x(4);

awg=[0.00797]*10^-6; %awg of chosen wire
Vbus=8.2;%V
CoreDens = 8.74e3; %kg/m^3
CuDens=8.93e3; %kg/m^3
CuRes=1.55*10^-8; %Ohm-meter, Resistivity
Wres= CuRes/awg;
mu_r=2000;
l_w=N*pi*2*r;
R=Wres*l_w;% + Rseries;

Nd = 4*(log(l./r)-1)./((l./r).^2 - 4*log(l./r));
% M_total = -(r*Vbus/2/Wres .*  (1+(mu_r-1)./(1+(mu_r-1).*Nd)))
M_total = pi*r^2*N*(Vbus/R)*(1+(mu_r-1)/(1+(mu_r-1)*Nd))


mass=CoreDens*pi*r.^2.*l+awg*l_w*CuDens
P = Vbus^2/R
l_w=N*pi*2*r
R=Wres*l_w;

end
```

```
%% Iron Core Magnetorquers for CADRE
% Duncan Miller
clear all
close all
clc

aw=[0.205 0.129, 0.102, 0.0810, 0.0642, 0.0509, 0.0404, 0.0320, 0.0254, 0.0201, 0.0127, 0.00797, 0.00501]*10^-6; %%%%%% American Wire Gauges in mm^2
gage=[24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38,40]; %Wire Gauge numbers
CuDens=8.93e3;                    %kg/m^3 Density of Copper
CuRes0=1.55*10^-8;                %Ohm-meter, Resistivity of copper
CuTempRes=3.9e-3;                 %K^-1, Temp coefficient of copper
T=0;                              %change in Temperature from T=293K (20C)
CuRes=CuRes0*(1+dT*CuTempRes);    %Temperature dependent Resistivity of copper
Vbus =8.2;                        %Bus supply voltage (from V_batt)
Pmax = 0.75;                      %W Max power allowable
CoreDens = 8.74e3                 %kg/m^3  Density of iron core

% Radius
% rmin=0.005;
% rinc=0.001;
% rmax=0.03;
% lmin=0.02;
% linc=0.001;
% lmax=0.1;
% zmin=0;
% zmax=2;

rmin=3e-3;
rinc=1e-4;
rmax=6e-2;
lmin=3e-3;
linc=0.001;
lmax=0.06;
zmin=0;
zmax=0.6;

awg=aw(12);

[r, l]=meshgrid(rmin:rinc:rmax, lmin:linc:lmax);
mu_r= 2000; % relative permeability of the core material


Rmin = Vbus^2/Pmax;
Wres= CuRes/awg;
l_w = Rmin/Wres; %Total length of wire
mass=awg*l_w*CuDens
N = l_w/2/pi./r;
```

```
Nd = 4*(log(l./r)-1)./((l./r).^2 - 4*log(l./r));
M_total = r*Vbus/2/Wres .*  (1+(mu_r-1)./(1+(mu_r-1).*Nd));
mass=CoreDens*pi*r.^2.*l+awg*l_w*CuDens;

surf(r,l,M_total, 'EdgeColor', 'none')
axis([rmin rmax lmin lmax zmin zmax])
xlabel('Core Radius (m)')
ylabel('Core Length (m)')
zlabel('Magnetic Dipole Moment (A-m^2)');
%{
figure(2)
surf(r,l,N, 'EdgeColor', 'none')
axis([rmin rmax lmin lmax 100 5000])
xlabel('Core Radius (m)')
ylabel('Core Length (m)')
zlabel('Number of Turns (A-m^2)') %%%%%% I think you want to change this unit to (-) %%%%%%

figure(3)
surf(r,l,mass, 'EdgeColor', 'none')
axis([rmin rmax lmin lmax 0 0.3])
xlabel('Core Radius (m)')
ylabel('Core Length (m)')
zlabel('Mass (kg)')

%}
%% Calculations of Properties
clear all
clc
N=[13550] %count
awg=[0.00797]*10^-6 %awg of chosen wire
l= [5] *10^-2 %m
r= [0.73]*10^-2 %m
l_w=N*pi*2*r
Vbus=8.2;%V
CoreDens = 8.74e3; %kg/m^3
CuDens=8.93e3; %kg/m^3
CuRes=1.55*10^-8; %Ohm-meter, Resistivity
Wres= CuRes/awg;
mu_r=2000;


mass=CoreDens*pi*r.^2.*l+awg*l_w*CuDens
Nd = 4*(log(l./r)-1)./((l./r).^2 - 4*log(l./r))
M_total = r*Vbus/2/Wres .*  (1+(mu_r-1)./(1+(mu_r-1).*Nd))
R=Wres*l_w; %Total length of wire
P=Vbus.^2/R
```

```
function [ C, Ceq ] = mycon( x )
%mycon University of Michigan
```

```
%   These are the inequality constraints for the CADRE torquer optimization
%   script

%Free variables are the turn count, core radius and core length
N=x(1);
r=x(2);
l=x(3);

%A series resistance was considered to reduce current
%Rseries = x(4);

%Known constants
awg=[0.00797]*10^-6;   %awg of chosen wire
Vbus=8.2;              %V
CoreDens = 8.74e3;     %kg/m^3
CuDens=8.93e3;         %kg/m^3
CuRes=1.55*10^-8;      %Ohm-meter, Resistivity
Wres= CuRes/awg;       %Ohm/meter
mu_r=2000;             %Relative Permeability
Nd = 4*(log(l./r)-1)./((l./r).^2 - 4*log(l./r));

%Wire length
l_w=N*pi*2*r;
R=Wres*l_w;%+ Rseries;

%Inequality Constraints
C(1)= CoreDens*pi*r.^2.*l+awg*l_w*CuDens-0.03; %mass is less than 30g
C(2)= Vbus.^2/R-0.2;      % power is less than 200mW
C(3)= -x(1)+10;           %Number of turns is something reasonable
C(4)= x(1)-10000;
C(5)= -x(2)+3e-3;         %Radius is something reasonable
C(6)= -x(3)+5e-2;         %Length is something reasonable
C(7) = r - l;             %Keep the design space in the feasible quadrant
%C(7) = -pi*r^2*N*(Vbus/R)*(1+(mu_r-1)/(1+(mu_r-1)*Nd)) + 0.05;

Ceq= [];%-x(3)+5e-2;

end
```